As a result of the vanishing of the covariant derivatives of the metric tensor, the covariant derivative of the determinant $g$ of $g_{\mu\nu}$ also vanishes,

$$\nabla_\alpha g = 0. \tag{2.7.24}$$

4 The covariant derivative of the Kronecker delta tensor is equal to zero,

$$\nabla_\alpha \delta^\mu_\nu = 0. \tag{2.7.25}$$

The verification of this equation is by a direct calculation:

$$\begin{aligned}\nabla_\alpha \delta^\mu_\nu &= \frac{\partial \delta^\mu_\nu}{\partial x^\alpha} + \Gamma^\mu_{\alpha\rho}\delta^\rho_\nu - \Gamma^\rho_{\alpha\nu}\delta^\mu_\rho \\ &= \Gamma^\mu_{\alpha\nu} - \Gamma^\mu_{\alpha\nu} = 0.\end{aligned}$$

5 The covariant derivative of a scalar function $\phi(x)$ is equal to its partial derivative,

$$\nabla_\alpha \phi(x) = \frac{\partial \phi(x)}{\partial x^\alpha}. \tag{2.7.26}$$

From the above rules it follows, since the covariant derivative of the metric tensor vanishes, that raising and lowering the indices of tensors is not affected by the operation of covariant differentiation. For example,

$$\nabla_\alpha V^\beta = \nabla_\alpha\left(g^{\beta\gamma} V_\gamma\right) = g^{\beta\gamma}\nabla_\alpha V_\gamma. \tag{2.7.27}$$

**Some Useful Formulas**

To conclude this section we derive some useful formulas which are related to the concept of covariant differentiation discussed above.

The covariant divergence of a vector $V^\mu$ is given by

$$\nabla_\mu V^\mu = \frac{\partial V^\mu}{\partial x^\mu} + \Gamma^\mu_{\alpha\mu} V^\alpha. \tag{2.7.28}$$

Using now Eq. (2.6.20), we obtain, for the expression of divergence the following:

$$\nabla_\mu V^\mu = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\, V^\mu\right). \tag{2.7.29}$$